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A new class of non-topological solitons

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Abstract. We construct a new class of non-topological solitons (NTSs) with scalar self-interaction term $\kappa\phi^4$. Because of the scalar self-interaction, there is a maximum size for these objects. There exists a critical value κ_{crit} for the coupling. For $\kappa > \kappa_{\text{crit}}$ there are no stable non-topological solitons. In the thin-walled limit, we show the explicit solutions of NTS with scalar self-interaction and/or gauge interaction. In the case of gauged NTS, soliton becomes a superconductor.

1. Introduction

Non-topological solitons (NTSs) are stable solutions of the classical field theory [1], where the field is confined to a finite region in space and there exists a conserved Noether charge. A potential containing only up to quartic terms in a complex scalar field does not admit NTS solutions. To get such solutions one has to add new terms to the potential. If the complex scalar field is a fundamental field, then in order to have a renormalizable theory, one can introduce an additional real scalar field σ with, for instance, a potential term

$$U(\sigma)=\tfrac{1}{4}\lambda(\sigma^2-\sigma_0^2)^2 \tag{1}$$

$\sigma=0$ corresponds to the false vacuum state, whereas $\sigma=\sigma_0$ is the normal vacuum. Instead, if the complex scalar field ϕ is purely phenomenological, and therefore renormalizability is no longer required, one can include higher power terms in the potential. The simplest self-interaction potential for ϕ with a degenerate vacuum is

$$U(\phi\phi^*)=\frac{\lambda^2(\phi\phi^*)^3}{6\mu^2}-\frac{(\phi\phi^*)^2}{4}+\frac{\mu^2\phi\phi^*}{2} \tag{2}$$

where μ is a mass parameter, λ is a dimensionless parameter, and $|\phi|=0$ corresponds to the normal vacuum. In the past few years, non-topological solitons have been investigated widely in the explanation of soliton stars [2], cosmic neutrino balls [3], quark nuggets [4], Bose liquid [5], structure of hadrons [6], and a scenario for producing them in a phase transition in the early universe has been considered [7]. The simplest example of a NTS is the Q-ball that can appear in a $U(1)$ invariant theory with a

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complex scalar field that has nonlinear self-interactions. Furthermore, the gauged Q-balls have been studied in a local $U(1)$ theory by Lee *et al* [8], in which they have considered the non-renormalizable potential (2). This work provides the possibility for understanding how NTSs might arise in realistic gauge theories such as electromagnetism, or unified theories. Recently, we have also studied the existence and stability of the non-topological fermion string [9] and the classical solutions of the non-topological soliton in a renormalizable local $U(1)$ theory [10]. In this paper, we extend the work of Freidberg *et al* [11] on non-topological solitons. Specifically, we study the effects of adding a quartic scalar self-coupling and of coupling the conserved charge to a gauge field. In the thin-walled limit, we show the explicit solutions of the non-topological soliton with scalar self-interaction and/or gauge interaction. In the gauged case, the soliton becomes a superconductor. For large Q, there exist $Q_{\max}$ and $Q_{\min}$ in couplings less than their critical values.

It was proposed by Friedberg *et al* [11] that the simplest renormalizable field theory that admits *non-topological solitons is composed of two scalar fields: the real scalar* σ and the complex scalar ϕ. Let us consider the following Lagrangian, which is invariant under the discrete symmetry $\sigma\to-\sigma$ and the global $U(1)$ symmetry $\phi\to e^{i\alpha}\phi$:

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu\sigma)^2+|D_\mu\phi|^2-V(\phi,\sigma)-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{3}$$

where $D_\mu=\partial_\mu-ieA_\mu$ and

$$V(\phi,\sigma)=\tfrac{1}{4}\lambda(\sigma^2-\sigma_0^2)^2+\kappa|\phi|^4+\frac{\mu^2}{\sigma_0^2}\sigma^2|\phi|^2. \tag{4}$$

The spherically symmetric NTS solutions for the above Lagrangian were first studied by Friedberg *et al* [11] for the special case $\kappa=0$ and $e=0$. In this paper, we are going to study the situation for $\kappa\neq0$ and/or $e\neq0$. Here we are only interested in the case when $\sigma_0^2>0$. Therefore, the discrete symmetry is broken spontaneously in the ground state with $\langle\sigma\rangle=\mp\sigma_0$. The $U(1)$ symmetry is still intact. As a result, there is a Noether current

$$J_\mu=-i(\phi^*D_\mu\phi-\phi D_\mu\phi^*) \tag{5}$$

and a corresponding conserved charge

$$Q=\int d^3\gamma\, J_0 \tag{6}$$

which is the necessary condition for the stability of a non-topological soliton. For large Q, it is characterized by an interior false vacuum region with $\sigma=0$, surrounded by a thin domain wall where σ rapidly approaches its ground state value $\sigma=\sigma_0$. In the soliton interior, the potential energy density in σ is balanced by the pressure of the massless ϕ charges, which are confined by the mass gap μ at the domain wall. As a simple consequence of the symmetry breaking for the $e\neq0$ case, the gauge field is massive inside and the soliton is a $U(1)$ superconductor.

2. Qualitative properties of soliton

We begin by deriving the solitons for the $\kappa\neq0$ and $e\neq0$ case as a natural extension of the derivation in [11]. We expect the solitons to be stable as long as their self-energy is much smaller than the other energies. Consider a coherent configuration of ϕ, σ and A_μ with a given electric charge eQ. The lowest energy solutions are of the spherically symmetric form [1, 10]

$$\begin{aligned} &\phi(\boldsymbol{r}, t) = \mathrm{e}^{\mathrm{i}\omega t}\,\phi(r)/\sqrt{2} \\ &\sigma(\boldsymbol{r}, t) = \sigma(r) \\ &A_\mu(r) = 0 \qquad (\mu \neq 0) \qquad A_0 = [\omega - g(r)]/e \end{aligned} \tag{7}$$

where we assume $\omega > 0$ for definiteness. The lowest energy state will have no electric currents and therefore no magnetic fields. The spatial components of the gauge potential are zero as there is no magnetic field. We choose a boundary condition $A_0(r) \to 0$ as $r \to \infty$.

The Lagrangian for the configuration described above is

$$L = 4\pi \int r^2\,\mathrm{d}r \left[\frac{1}{2} g^2\phi^2 - \frac{1}{2}\phi'^2 - \frac{1}{2}\sigma'^2 + \frac{1}{2e^2} g'^2 - V(\phi(r), \sigma(r))\right] \tag{8}$$

where prime denotes $\mathrm{d}/\mathrm{d}r$. By varying (8) with respect to ϕ, σ and g at fixed ω, we find the equations of motion:

$$\frac{1}{r}(r\phi)'' = \kappa\phi^3 + \frac{\mu^2}{\sigma_0^2}\sigma^2\phi - g^2\phi \tag{9}$$

$$\frac{1}{r}(rg)'' = e^2\phi^2 g \tag{10}$$

$$\frac{1}{r}(r\sigma)'' = \lambda\sigma(\sigma^2 - \sigma_0^2) + \frac{\mu^2}{\sigma_0^2}\phi^2\sigma. \tag{11}$$

The energy functional for the solution (7) can be written as

$$E = 4\pi \int r^2\,\mathrm{d}r \left[\frac{1}{2} g^2\phi^2 + \frac{1}{2}\phi'^2 + \frac{1}{2}\sigma'^2 + \frac{1}{2e^2} g'^2 + \frac{1}{4}\lambda(\sigma^2 - \sigma_0^2)^2 + \frac{1}{4}\kappa\phi^4 + \frac{1}{2}\frac{\mu^2}{\sigma_0^2}\sigma^2\phi^2\right]. \tag{12}$$

The soliton charge is

$$Q = 4\pi \int r^2\phi^2 g\,\mathrm{d}r. \tag{13}$$

To gain insight into the NTS solutions for $\kappa \neq 0$ and $e \neq 0$, we show some qualitative properties of the soliton solution. From (10) and (13) we have

$$g(r) \to \omega - \frac{e^2 Q}{4\pi r} \tag{14}$$

as $r \to \infty$. Furthermore, ϕ approaches zero and σ approaches the constant σ_0 for large r, and (9) can be reduced to

$$\frac{1}{r}(r\phi)'' + (\omega^2 - \mu^2)\phi = 0. \tag{15}$$

For the approximate equation (15), we have a solution as follows

$$\phi(r) = \phi_0 \exp(-\sqrt{\mu^2 - \omega^2}\,r)/r. \tag{16}$$

Clearly, a necessary condition for the existence of a solution is $\omega < \mu^2$. Additionally, for the soliton to be well-behaved at the origin, ϕ', σ' and g' should approach zero at least faster than r for $r \to 0$. By using the asymptotic behaviour of ϕ, σ and g the energy functional can be written as

$$E = \tfrac{1}{2}\omega Q + 4\pi \int r^2\, \mathrm{d}r[\tfrac{1}{2}\phi'^2 + \tfrac{1}{2}\sigma'^2 + V(f, \sigma)]. \tag{17}$$

Recall that *non-topological solitons are quantum mechanically stable if they are the* lowest energy configuration of the fixed charge. For the $\kappa = 0$ and $e = 0$ case. In [11] it was shown that $E \simeq \frac{4}{3}\sigma_0\lambda^{1/4}Q^{3/4}$ for large Q so that for $Q > Q_s$ the lowest energy state is favoured over the free particle one ($E_{\text{free}} = \mu Q$). For the $\kappa \neq 0$ case, we expect that the energy will be increased over the $\kappa = 0$ case due to ϕ^4 self-interaction becoming more important as Q becomes large. Therefore, if $\partial E/\partial Q > \mu$, we must consider that some charge can be put into the interior region of the soliton and some can be put in free particles. As discussed above, there exists a $Q_{\min}$ and a $Q_{\max}$, so that when $Q > Q_{\min}$ the nontopological soliton is quantum mechanically stable, when $Q > Q_{\max}$ the lowest energy state of the system is composed of a nontopological soliton with charge $Q_{\max}$ together with free particles carrying charge $Q - Q_{\max}$. In the $\kappa = 0$ and $e = 0$ theory, since $Q_{\max} \to \infty$, the condition $Q_{\min} < Q_{\max}$ is always satisfied.

For the $e \neq 0$ case, we have a similar discussion for the $Q_{\max}$ case because the energy will be increased over the $e = 0$ case due to Coulomb repulsion, with Coulomb energy becoming more important as Q becomes large. For the $\kappa \neq 0$ and $e \neq 0$ case, we must consider both the scalar self-energy and Coulomb energy. In the latter two cases, there exists also a maximum charge $Q_{\max}$ such that $Q > Q_{\max}$, a non-topological soliton with charge $Q_{\max}$ plus $Q - Q_{\max}$ free particles will be the lowest energy state for the system.

3. Thin-walled soliton with self-interaction $\kappa\phi^4$

In the $\kappa \neq 0$ and $e = 0$ case, the equations of motion (9)–(11) are reduced to

$$\frac{1}{r}(r\phi)'' = \kappa\phi^3 + \frac{\mu^2}{\sigma_0^2}\sigma^2\phi - \omega^2\phi \tag{18}$$

$$\frac{1}{r}(r\sigma)'' = \lambda\sigma(\sigma^2 - \sigma_0^2) + \frac{\mu^2}{\sigma_0^2}\phi^2\sigma. \tag{19}$$

In principle, the NTS solution can be constructed by choosing suitable values for the parameters ω, κ, μ and λ, as well as making use of the boundary conditions $\phi(\infty) = 0$, $\sigma(\infty) = 1$ and $\phi'(0) = 0$, $\sigma'(0) = 0$. Then the numerical solution of the non-topological soliton can be easily obtained from (18) and (19). In this paper, however, we would like to pursue an analytic solution. For this, we select the thin-wall limit and make use of the following spherically symmetric test functions:

$$\phi(r) = \begin{cases} f(r) & r \leqslant R \\ 0 & r > R \end{cases} \tag{20}$$

$$\sigma(r) = \begin{cases} h(r) & r \leqslant R \\ 1 - \exp[-(r-R)/L] & r > R. \end{cases} \tag{21}$$

There are two length parameters R and L, where R is the radius of the soliton, determined from the first zero of $f(r)$, and L is the thickness of the soliton wall which separates the internal false vacuum from the external real vacuum. When Q is large (the thickness of the soliton wall is much less than the soliton radius), the energy associated with the wall can be neglected. Therefore, in the thin-wall limit $L \to 0$, $\sigma(r)$ can be considered as a step function. The equations of motion are reduced to

$$\frac{1}{r}(rf)'' = \kappa f^3 - \omega^2 f. \tag{22}$$

This equation can be solved explicitly by the series in the form

$$f = \frac{a_0 \sin \omega r}{r} + \kappa a_0^3 \omega \sum_{k=1}^{\infty} \frac{a_k (\omega r)^{2k}}{(2k+1)!} \tag{23}$$

where

$$\begin{aligned} &a_1 = 1 \qquad a_2 = 3\kappa a_0^2 - 4 \qquad a_3 = 19\kappa^2 a_0^4 - 35\kappa a_0^2 + 17 \\ &a_4 = \tfrac{619}{3}\kappa^3 a_0^6 - 488\kappa^2 a_0^4 + 366\kappa a_0^2 - \tfrac{256}{3} \ldots . \end{aligned} \tag{24}$$

The convergence radius of the series (23) is dependent on coupling constant κ and the parameter $a_0 = f(0)$. Further, when $\kappa a_0^2 < 7.168$, we find the convergence radius $> 1/\omega$. By using (23) it is now straightforward to solve numerically (22). The results for different values of the coupling constant κ are given in figure 1. The function $f(r)$ becomes a constant solution for the $\kappa = 1$ case. By using the Runge–Kutta method, the 'analytic' solutions are corroborated.

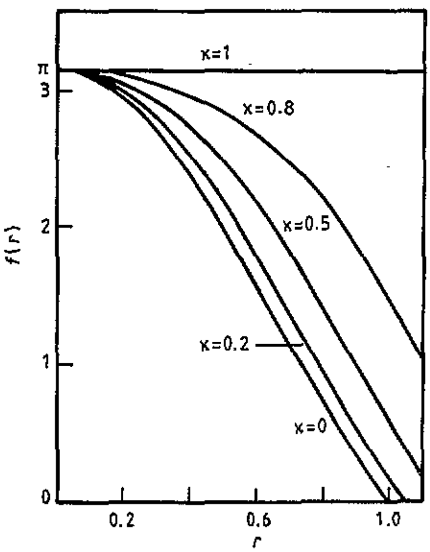

Figure 1. The solution $f(r)$ in units of $f(0)/\pi$ as a function of the radius r in units of π/ω for different values of the coupling constant κ.

In the weak coupling situation of $\kappa \ll 1$, we have

$$f \approx \frac{a_0 \sin \omega r}{r}. \tag{25}$$

Substituting (25) into the energy functional, we obtain

$$E = \omega Q + \frac{\pi^4 \lambda \sigma_0^4}{3\omega^3} + \frac{\kappa Q^2 \omega I}{4\pi^2} \tag{26}$$

where

$$I = \frac{1}{\pi} \int_0^\pi \frac{\sin^4 x}{x^2} \mathrm{d}x \approx \frac{6}{25}. \tag{27}$$

For a fixed value of Q, the radius R of the soliton can be determined from the minimum of the energy:

$$R_{\min} \simeq \frac{1}{\sigma_0} \left[\frac{1}{\lambda} \left(Q + \frac{I\kappa}{4\pi^2} Q^2 \right) \right]^{1/4}. \tag{28}$$

At the radius $R_{\min}$, the energy of the nontopological soliton is

$$E \simeq \tfrac{4}{3} \pi \sigma_0 \lambda^{1/4} Q^{3/4} \left(1 + \frac{\kappa I}{4\pi^2} Q \right)^{3/4}. \tag{29}$$

According to (29) and $E < \mu Q$, we have

$$\left(Q + \frac{4\pi^2}{\kappa I} \right)^3 < \frac{81 \pi^2 Q}{4 \kappa^3 I^3 \lambda} \left(\frac{\mu}{\sigma_0} \right)^4. \tag{30}$$

The value of Q depends on κ and μ/σ_0, which are model-dependent parameters. It is easy to derive that

$$Q_{\max} = \frac{3\mu^2 \pi}{\kappa^{3/2} I \sigma_0^2} \sqrt{\frac{3}{I\lambda}} \cos\left[\frac{1}{3} \left(\pi - \cos^{-1} \frac{4\pi \sigma_0^2 \kappa^{1/2}}{\mu} \sqrt{\frac{I\lambda}{3}} \right) \right] - \frac{4\pi^2}{\kappa I} \tag{31}$$

$$Q_{\min} = \frac{3\mu^2 \pi}{\kappa^{3/2} I \sigma_0^2} \sqrt{\frac{3}{I\lambda}} \cos\left[\frac{1}{3} \left(\pi + \cos^{-1} \frac{4\pi \sigma_0^2 \kappa^{1/2}}{\mu} \sqrt{\frac{I\lambda}{3}} \right) \right] - \frac{4\pi^2}{\kappa I}. \tag{32}$$

There exists a critical value κ_{crit} for the coupling κ:

$$\kappa_{\text{crit}} = \frac{3}{16\pi^2 I \lambda} \left(\frac{\mu}{\sigma_0} \right)^4. \tag{33}$$

When $\kappa < \kappa_{\text{crit}}$, we have $Q_{\min} < Q_{\max}$. In other words, when Q satisfies $Q_{\min} < Q < Q_{\max}$, the non-topological soliton solution of (18) and (19) is stable. When $\kappa > \kappa_{\text{crit}}$, stable solutions for the non-topological solitons do not exist.

4. Gauged thin-walled soliton

In the $\kappa=0$, $e\neq 0$ case, we consider spherically symmetric trial functions (20), (21) and

$$g(r)=\begin{cases}\tilde{g}(r) & r\leqslant R\\ \omega-\dfrac{e^2Q}{4\pi r} & r>R.\end{cases} \tag{34}$$

In the thin-wall limit, the equations of motion (9)–(11) are reduced to

$$\frac{1}{r}(rf)''+f\tilde{g}^2=0 \tag{35}$$

$$\frac{1}{r}(r\tilde{g})''-e^2f^2\tilde{g}=0. \tag{36}$$

One can find the power-series solutions for f and $\tilde{g}$:

$$f=a\sum_{k=0}^{\infty}\frac{(-1)^k f_{2k}\gamma^{2k}}{(2k+1)!} \tag{37}$$

$$\tilde{g}=b\sum_{k=0}^{\infty}\frac{g_{2k}\gamma^{2k}}{(2k+1)!}. \tag{38}$$

The recursion formula of the coefficients f_{2k} and g_{2k} can be expressed as follows:

$$f_{2m+2}=(2m+1)!\sum_{n=0}^{m}\sum_{k=0}^{n}\frac{(-1)^n g_{2k}}{(2k+1)!}\cdot\frac{g_{2(n-k)}}{(2n-2k+1)!}\cdot\frac{f_{2(m-n)}}{(2m-2n+1)!}b^2 \tag{39}$$

$$g_{2m+2}=(2m+1)!\sum_{n=0}^{m}\sum_{k=0}^{n}\frac{(-1)^n f_{2k}}{(2k+1)!}\cdot\frac{f_{2(n-k)}}{(2n-2k+1)!}\cdot\frac{g_{2(m-n)}}{(2m-2n+1)!}e^2a^2. \tag{40}$$

The function $f(r)$ may be written as a sum of two terms, the first part independent of e, the second part dependent upon e:

$$f(\gamma)=\frac{a}{br}\sin br-2e^2a^3b^2\gamma^4/5!+\left(\frac{16}{3}e^4a^5b^2-\frac{38}{3}e^2a^3b^4\right)r^6/7!-\dots. \tag{41}$$

In the weak coupling situation, we have

$$f=\begin{cases}\pi\sqrt{\dfrac{Q}{2}}\sin\left(\omega-\dfrac{e^2Q}{4\pi R}\right)r/r & r\leqslant R\\ 0 & r>R\end{cases} \tag{42}$$

$$g=\begin{cases}\omega-\dfrac{e^2Q}{4\pi R} & r\leqslant R\\ \omega-\dfrac{e^2Q}{4\pi r}, & r>R\end{cases} \tag{43}$$

$$\sigma=\begin{cases}0, & r\leqslant R\\ \sigma_0, & r>R.\end{cases} \tag{44}$$

Substituting (42)–(44) into the energy functional, the energy of the NTS at the radius $R_{\min}$ is

$$E \simeq \tfrac{4}{3}\pi\sigma_0\lambda^{1/4}Q^{3/4}\left(1+\frac{e^2 Q}{8\pi^2}\right)^{3/4}. \tag{45}$$

According to (45) and $E<\mu Q$, we have

$$\left(Q+\frac{8\pi^2}{e^2}\right)<\frac{162\pi^2 Q}{e^6\lambda}\left(\frac{\mu}{\sigma_0}\right)^4. \tag{46}$$

From (46) we have

$$Q_{\max}=\frac{3\mu^2\pi}{2}\sqrt{\frac{3}{2\lambda}}\cos\left[\frac{1}{3}\left(\pi-\cos^{-1}\frac{2\pi\sigma_0^2 e}{\mu^2}\sqrt{\frac{2\lambda}{3}}\right)\right]-\frac{8\pi^2}{e^2} \tag{47}$$

$$Q_{\min}=\frac{3\mu^2\pi}{2}\sqrt{\frac{3}{2\lambda}}\cos\left[\frac{1}{3}\left(\pi+\cos^{-1}\frac{2\pi\sigma_0^2 e}{\mu^2}\sqrt{\frac{2\lambda}{3}}\right)\right]-\frac{8\pi^2}{e^2}. \tag{48}$$

There exists a critical value of e above which there is no solution to the equation defining $Q_{\max}$ and $Q_{\min}$. From (47) and (48), it is easy to find that

$$e_{\mathrm{crit}}^2=\frac{3}{8\pi^2\lambda}\left(\frac{\mu}{\sigma_0}\right)^4. \tag{49}$$

We see that the NTSs can occur when $e<e_{\mathrm{crit}}$. As a consequence of the symmetry, the gauge field is massive inside and soliton is a $U(1)$ superconductor.

5. General thin-walled soliton

Now we discuss general case with $\kappa\neq 0$ and $e\neq 0$. By using trial functions (20), (21) and (34), the thin-walled solution can be written as follows:

$$\phi=\frac{a}{\sqrt{2}}\left(\sum_{k=0}^{\infty} f_k r^{2k}\right)\theta(R-r) \tag{50}$$

$$\sigma=\sigma_0\theta(r-R) \tag{51}$$

$$A_\mu=\delta_{\mu 0}\left[\omega-\left(b\sum_{k=0}^{\infty} g_k r^{2k}\right)\theta(R-r)+\left(\omega-\frac{e^2 Q}{4\pi r}\right)\theta(r-R)\right]\Big/ e \tag{52}$$

where $\theta(x)$ is a step function, R is the radius of the soliton and

$$f_0=1 \qquad g_0=1 \qquad f_1=\frac{1}{3!}(\kappa-b^2) \qquad g_1=\frac{1}{3!}e^2a^2$$

$$f_2=\frac{1}{5!}(3\kappa^2-4\kappa b^2+b^4-2e^2a^2b) \qquad g_3=\frac{1}{5!}e^2a^2(2\kappa-2b^2+e^2a^2)\ldots. \tag{53}$$

The recursion formula of the coefficients f_k and g_k can be expressed as follows:

$$f_k=\frac{1}{2k(2k+1)}\sum_{l+m+n=k-1}(\kappa f_l f_m f_n-b^2 f_l g_m g_n) \tag{54}$$

$$g_k=\frac{e^2a^2}{2k(2k+1)}\sum_{l+m+n=k-1}f_l f_m g_n. \tag{55}$$

In the weak coupling case, using similar process to that above, we obtain the energy of the non-topological soliton as

$$E\simeq\tfrac{4}{3}\pi\sigma_0\lambda^{1/4}Q^{3/4}\left[1+\frac{IQ}{4\pi}\left(\kappa+\frac{e^2}{2I}\right)\right]^{1/4} \tag{56}$$

at the radius $R_{\min}$, where I is defined in (27). We can define an effective coupling constant κ_{eff}:

$$\kappa_{\text{eff}}=\kappa+\frac{e^2}{2I} \tag{57}$$

then (56) is as (29). Therefore, there exists also a critical value in the general case:

$$\left(\kappa+\frac{e^2}{2I}\right)_{\text{crit}}=\frac{3}{16\pi^2 I\lambda}\left(\frac{\mu}{\sigma_0}\right)^{\mu}. \tag{58}$$

However, the general thin-walled soliton is a superconductor, in contrast with the same problem in $\kappa\phi^4$ theory.

6. Discussion

The theoretical studies on boson stars are in a preliminary stage and hopefully future observations in particular physics will tell us whether such objects can exist and what their relevance for cosmology might be [12]. The simplest model, first discussed by Feinblum and MaKinley [13], Kaup [14], and Ruffini and Bonazzola [15], consists of a massive free complex scalar field. The order of magnitude of the maximal mass is $M_{\max}\approx M_{\text{pl}}^2/m$, in which m is mass of the scalar field. In a series of papers Lee and his colleagues [16] extended the results of Ruffini and Bonazzola [15]. In particular, they considered excited solutions of the coupled Einstein–Klein–Gordon equations with nodes for the radial dependence of the scalar field, which were called soliton stars. The critical mass of a soliton star is typically of order M_{pl}^4/m^3. By using the new class of non-topological solitons, it is interesting to notice that another class of boson stars can be suggested.

We showed in the above, a new class of non-topological solitons and pointed out the existence of a critical value for the coupling constants. When $\kappa+e^2/2I$ is larger than the critical value, there does not exist any stable non-topological soliton. We make three simple comments. (i) In the $\kappa \to 0$ or $e \to 0$ limit, from (31) and (32) or (47) and (48) we obtain $Q_{\max} \to \infty$ and $Q_{\min} \to \lambda (4\pi\sigma/3\mu)^4$, which agrees with the study of Lee and his colleagues [11]. (ii) The Lagrangian (3) has the discrete symmetry $\sigma \to -\sigma$ spontaneously broken in the ground state with $\langle\sigma\rangle \to \mp\sigma_0$. If $\sigma_0 > 1$ MeV, this will lead to the well-known problem of domain walls, which breaks the discrete symmetry explicitly. Since this additional term will be very tiny, it will not affect the study made in this paper. (iii) Furthermore if we introduce a realistic model [17] we arrive at a similar conclusion. This separate work will be reported in future.

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